THE ROYAL AERONAUTICAL SOCIETY

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS.

MONTHLY NOTICES,

JULY, 1942.

General Board of the National Physical Laboratory.

The following members have been nominated to represent the Society on the General Board of the National Physical Laboratory:—

Mr. Griffith Brewer, Hon. F.R.Ae.S. (President of the Society).

Mr. W. C. Devereux, F.R.Ae.S. (Vice-President of the Society).

Birthday Honours.

The following members received Honours in the recent Birthday Honours List:—

Knights Bachelor.

Mr. C. R. Fairey, M.B.E., Fellow.

Mr. F. Handley Page, C.B.E., Fellow.

Mr. R. A. Watson Watt, C.B., F.Inst.P., M.Inst.E.E., Fellow.

C.B.

Mr. W. P. Hildred, O.B.E., M.A. (Companion Member).

C.B.E.

Group Captain W. Helmore, Ph.D., M.Sc., Fellow.

O.B.E.

J. Lankester Parker, Associate Fellow.

D.S.O.

Wing Commander D. C. T. Bennett, Fellow.

Society Meeting.

It is proposed to hold a meeting of the Society to discuss methods of Performance Calculation. The meeting will be arranged if a sufficient number of members express a wish for it. Will those members who would like to attend write in, therefore, to the Secretary.

Meteorological Research Committee.

The Secretary of State for Air has appointed a Meteorological Research Committee to assist in the carrying out of meteorological investigations. The Chairman of this Committee is Professor S. Chapman, and the members are Professor D. Brunt, Dr. G. M. B. Dobson, Professor G. I. Taylor, the Director of the Meteorological Office, the Director of Scientific Research (Ministry of Aircraft Production), the Director of the Naval Meteorological Service and representatives of the Air Staff and Civil Aviation.

At present the Committee will naturally be concerned chiefly with problems directly concerned with the war effort, and such work for the time being must

remain secret, though any incidental results not likely to be of service to the enemy may be published.

The Committee will welcome contact and co-operation with University departments or other institutions engaged on work that bears on Meteorology.

Associate Fellowship New Syllabus Postponement.

Notice was given some two years ago of the provisions of the new syllabus, but the increasing difficulties created by the war situation has caused the Council to decide to defer putting the new syllabus into operation until at least six months after the cessation of hostilities. The present syllabus will therefore be continued until further notice.

Any student who has been preparing for subjects, however, which appear in the new syllabus and not the old, will have a paper set in that subject, so he will not be penalised.

Section A of the present syllabus or exemptions therefrom will continue to be recognised during the same period of postponement. It should be noted that in agreement with other Engineering Institutions, when the new Associate Fellowship Examinations are introduced that the Common Preliminary Examination or an exempting examination will be necessary. During the transitional period, either examination may be taken.

Associate Fellowship Examinations, May, 1942.

The following candidates were successful in the May Associate Fellowship Examinations:—

R. Allan	Pure Mathematics. Design (Aircraft) (First Place—Tie).
C. L. Brockenshaw ...	Aerodynamics. Aircraft Instruments—Theory and Design.
W. R. Bruce	Design (Aircraft).
Miss F. M. Buss ...	Air Transport (First Place).
A. Capella	Design (Aero Engines). Applied Mathematics (First Place—Tie). Theory of Machines (First Place—Tie).
G. Y. Carayan ...	Pure Mathematics.
J. F. Chenery	Pure Mathematics. Aircraft Materials (First Place).
O. Coker	Applied Mathematics. Strength of Aeronautical Materials and Structures.
E. Cook-Yarborough ...	Applied Mathematics. Design (Aircraft).
G. H. I. Cornwell ...	Theory of Machines.
R. F. Creasey ...	Aerodynamics (First Place). Applied Mathematics (First Place—Tie). Design (Aircraft) (First Place—Tie).
P. A. Curry	Applied Mathematics. Theory of Machines.
J. P. R. De Crabral ...	Applied Mathematics.
H. A. Davenall ...	Strength of Aeronautical Materials and Theory of Structures. Design (Aircraft).
Y. Desai	Design (Aero Engines). Applied Mathematics. Theory of Machines.

G. A. Dawson	...	Applied Mathematics.
R. Evans-Thomas	...	Applied Mathematics.
W. G. Gallia ...	...	Pure Mathematics.
R. H. Gapp ...	...	Design (Aero Engines). Theory of Machines. Applied Mathematics.
C. J. Goodwin	...	Design (Aircraft). Strength of Aeronautical Materials and Theory of Aircraft Structures (First Place). Pure Mathematics.
W. Hampton ...	...	Pure Mathematics. Strength of Aeronautical Materials and Theory of Structures. Design (Aircraft).
E. G. Havard	...	Aerodynamics.
R. Holl ...	...	Pure Mathematics. Aerodynamics. Strength of Aeronautical Materials and Theory of Structures.
L. A. Hopkins	...	Pure Mathematics. Design (Aircraft). Aerodynamics.
J. Kirby ...	...	Strength of Aeronautical Materials and Theory of Structures. Theory of Machines. Pure Mathematics.
R. Lawrence ...	...	Design (Aircraft).
R. M. Lloyd ...	...	Applied Mathematics. Design (Aircraft).
G. R. Matheson	...	Applied Mathematics. Design (Aero Engines).
H. C. Maynard	...	Aircraft Instruments—Theory and Design.
R. A. McGill ...	...	Applied Mathematics. Theory of Machines.
J. I. Mitchell ...	...	Theory of Machines. Pure Mathematics. Meteorology and its Application to Aeronautics (First Place).
S. T. Naismith	...	Air Transport.
P. Norman ...	...	Aerodynamics. Pure Mathematics. Design (Aircraft).
G. M. Parker ...	...	Theory of Internal Combustion Engines.
R. E. Phillips ...	...	Applied Mathematics. Design (Aircraft).
G. F. Pitts ...	...	Pure Mathematics. Aerodynamics. Strength of Materials and Theory of Structures.
W. A. E. Puntis	...	Design (Aircraft).
H. G. Radlett ...	...	Pure Mathematics. Strength of Aeronautical Materials and Theory of Structures. Design (Aircraft).

A. Robinson	Aerodynamics. Pure Mathematics (First Place). Meteorology and its Application to Aeronautics.
A. D. Ruscoe	Applied Mathematics. Strength of Aeronautical Materials and Theory of Structures.
G. W. Sapsworth-Griffith	Aircraft Instruments—Theory and Design.
P. Saunders	Strength of Aeronautical Materials and Theory of Structures. Design (Aircraft).
P. Steiner	Applied Mathematics.
L. G. St. Leger ...	Design (Aircraft). Strength of Aeronautical Materials and Theory of Structures.
D. I. Thompson ...	Applied Mathematics. Design (Aero Engines).
R. T. Weatherstone ...	Design (Aero Engines) (First Place).
R. W. Webster ...	Pure Mathematics. Strength of Aeronautical Materials and Theory of Structures. Design (Aircraft).
E. J. Weiss	Strength of Aeronautical Materials and Theory of Structures.
D. C. Whittley ...	Strength of Aeronautical Materials and Theory of Structures.
R. M. Wingate ...	Pure Mathematics. Strength of Aeronautical Materials and Theory of Structures. Design (Aircraft).
N. H. Wood	Applied Mathematics. Design (Aircraft).
B. Zutshi	Pure Mathematics. Design (Aero Engines).

Election of Members.

Associate Fellows.—Hubert John Allwright (from Student), Arthur Andrew Bage.

Associates.—Lawrence Arthur (from Student), Henry Nevill Rose Dale, Joseph Thomas Fellows, Michael Cadwalladr Grey, George Ingram (from Companion), Walter Jack Ruck (from Companion), Leonard Sydney Whicher, Albert Noel Wilcox, Peter John Wintle, Peter Thomas Woodcock, Joseph Woodward.

Graduates.—Antony Martin Edwards (from Student), Bertram Roland Grantham, Leo Anthony Mcquillin (from Student), Peter Fyfe Mouritz (from Student).

Students.—Robert Frank Brown, Ivor Macaulay Davidson, Jervois Campbell Firmin, Frederick Thomas Frazer, Sidney Lipman, Thomas George Preston, Owen Sydney Staveley Roberts, Raymond Frederick Sargent, Terence William Shreeve, Sidney John Smith.

Library Catalogue.

A catalogue of the books in the Library of the Society is now available to members, price 10/6 (postage 7d.).

Additions to Library.

Pamphlets in italics, with location reference following in brackets.
Books marked * or ** may not be taken out on loan.

*B.g.115.—Aircraft Identification, Parts 1 and 2 (together). Edited by Peter G. Masefield. Temple Press, Ltd. 1942. 4/-.

BB.b.96.—Airplane Lofting. By William Nelson. McGraw Hill. 1942. 12/6.

BB.b.99.—Practical Aircraft Stress Analysis. (3rd Ed.) By D. R. Adams. Sir Isaac Pitman and Sons. 1942. 8/6.

BB.e.25.—The Research Laboratories of High Duty Alloys, Ltd., Slough. 1940.

BB.f.46.—Aircraft Instruments: their Construction and Maintenance. By J. Riley. N.A.G. Press. 1942. 1/6.

*D.b.175.—International Air Transport and National Policy. By Oliver J. Lissitzyn. Council on Foreign Affairs, U.S.A. 1942. $5.00.

D.e.34.—*The Value of a Pound. By John E. Ayers. Society of Aeronautical Weight Engineers, U.S.A. (S.A.W.E. Paper No. 23.)* 1942. (Y.34.)

E.d.10.—Micrometers, Slide Gauges and Calipers. By A. W. Marshall and G. Gentry. Percival Marshall Co. (Undated.) 2/-.

E.d.11.—Practical Lessons in Metal Turning and Screwcutting. By Percival Marshall. Percival Marshall Co. (Undated.) 3/6.

E.d.12.—Gauges and Gauging. By R. Barnard Way. Percival Marshall Co. (Undated.) 2/-.

G.a.54.—Aircraft Sheet Metal Work. By Carl Norcross and James D. Quinn. McGraw Hill. 1942. 15/-.

G.a.55.—*Note on an X-Ray Investigation into Cold-Rolled Magnesium Alloy Sheet. By Leopold Fromer. Institute of Metals.* 1941. (PG.1/18.)

*G.b.41.—Hiduminium Technical Data. High Duty Alloys, Ltd. 1941.

G.e.A.79.—Hiduminium Anticorodal Alloy. High Duty Alloys, Ltd. (Undated.)

G.e.A.81.—Aluminium Alloys. By A. v. Zeerleder. (Translated by A. J. Field.) High Duty Alloys, Ltd. 1936.

G.e.A.82.—*The Routine Analysis of Aluminium Alloys. By Sydney Torrance. High Duty Alloys, Ltd.* 1942. (Y.13.c.A.)

G.e.A.83.—*Aluminium Alloys in Aircraft. By W. C. Devereux. High Duty Alloys, Ltd.* 1939. (Y.13.c.A.)

G.e.E.55.—*Special and Alloy Steels. By W. H. Hatfield. Reprint from "Metallurgia," April,* 1942. (Y.13.c.S.)

G.e.G.23.—Plywoods. By A. D. Wood and T. G. Linn. W. and A. K. Johnston. 1942. 25/-.

G.e.J.20.—Magnuminium. Magnesium Castings and Products, Ltd. (Undated.)

G.e.J.21.—*Radiography Applied to Magnesium Alloy Castings. By Philip M. Bailey. High Duty Alloys, Ltd. (Undated.)* (Y.13.c.M.)

G.f.27.—Aircraft Welding. By L. S. Elzea. McGraw Hill. 1942. 14/-.

L.c.23.—Air Pilot Training. By Bert A. Shields. McGraw Hill. 1942. 24/-.

**R.b.232.—*Experiments in Flying Machines. By Percy S. Pilcher. Dollard, Dublin.* 1897. (Autographed copy.)

S.b.111.—*R.A.F. Parade. Air Ministry Official Publication.* 1941. 1/-. PS.3.a.15.)

S.b.112.—*We speak from the Air. (R.A.F. Broadcasts.) H.M.S.O.*, 1942. 6*d*. (PS.3.a.16.)

S.d.77.—*Arming in the Air. Brig. Gen. P. R. C. Groves (Writer of Preface). Associated Newspapers, Ltd.* 1936. 1*d*. (PS.2/5a.)

UL.3.—*Institute of Physical and Chemical Research, Tokyo. Scientific Papers, Nos.* 1,036-1,038. 1941.

*UU.c.—*National Advisory Committee for Aeronautics, U.S.A. Technical Memoranda*:—

No. 1009.—Theoretical Solution of Profile Drag. By J. Pretsch. (From Jahrbuch 1938 der D.L.F.F.)

No. 1011.—Pressure Distribution on Wings in Reversed Flow. By A. Naumann. (From Jahrbuch 1938 der D.L.F.F.)

WB.12/22.—The Nickel Bulletin, Vol. XIV (1941).

WB.17/14.—Journal and Proceedings of the Institution of Electrical Engineers, Vol. 88 (1941).

The following books and pamphlets have been bequeathed to the Society from the library of the late Major D. H. Kennedy:—

B.a.254.—The Modern Aeroplane. (Diagrammatic pictures, issued by Shell-Mex Co.) Undated.

B.a.255-7.—*Pamphlets issued by Bristol Aeroplane Co. (about* 1933):—
Bristol Bulldog 2-seater Training Machine.
Bristol Bulldog 1933 Type.
Bristol Bulldog All-Steel Single-Seater Fighter. (Box-file Y.3.ii.B.)

**B.g.112.—*Official Air Ministry Publications.* (1918):—
D.A.I. 101 (F.S. Pub. 21.) Types of British Aeroplanes.
D.A.I. 109 (F.S. Pub. 63). Types of German Aeroplanes and Flying Boats.
D.A.I. 103 (F.S. Pub. 47). Types of German Aeroplanes.
D.A.I. 6 (F.S. Pub. 60). Types of British Seaplanes, Flying Boats and Ships' Aeroplanes.

**B.g.113-4.—A.I.D. Aeroplane Data Book, No. III, Vols. 1 and 2. 1917.

BB.b.97.—Metal Aircraft Construction. By Marcus Langley. Sir Isaac Pitman and Sons. 1937.

BB.b.98.—*Bristol Steel Aircraft Construction. Bristol Aeroplane Co. (Undated, about* 1933-1935.) (Y.8.)

BB.d.30.—Hydraulic Actuating Equipment for Aircraft. By R. H. Bound. Aircraft Components, Ltd. 1938.

D.a.B.81.—The Approach Towards a System of Imperial Air Communications. Air Ministry. H.M.S.O. 1926.

D.a.B.82.—Report on the Progress of Civil Aviation, 1929. Air Ministry. H.M.S.O. 1930.

D.b.171.—*"You're Welcome!" (Account of American and Canadian Tour, partly by Air.) Lawrence Wingfield. Privately Printed.* 1938. (PD.1-18a.)

D.b.172.—*The Air Way. By Harry Harper. Imperial Airways, Ltd. Undated (about* 1930). (PD.1/8a.)

D.b.173.—*Air Mail. G.P.O. Official Regulations.* 1931. (Y.6.a.5a.)

D.b.174.—*The Open Speedway of the Air. National Flying Services.* 1929. *(Advt. Booklet.)* (Y.6.b.4a.)

EE.a.81.—Handbook for the Installation, Running and Maintenance of Rolls-Royce "Kestrel" Aero Engines (Series IV to XII). Rolls-Royce, Ltd. 1937.

EE.a.82.—Gnome "Mono" Instruction Book No. 2. Gnome and Le Rhône Engine Co. (Undated.)

EE.a.83 and 84.—*Instruction Handbooks for the Maintenance of Wolseley Air-Cooled Radial Aero Engines (Aries, Mark I, II and III), (Scorpio, Mark I).* 1935. (PEE.1/13 and 14.)

EE.b.63.—*Bristol Mercury and Pegasus Engines.* 1932. *Bristol Aeroplane Co.* (Y.7.d.B.)

EE.f.50.—*Heavy Oil and Diesel Engines. By Charles Day.* (*Lecture before Institution of Mechanical Engineers, March* 16, 1930.) PEE.4.a.3.)

EE.j.33.—*The Thomas Transmission: an Electro-Mechanical Variable Speed Gear. By Hedley J. Thompson.* (*Lecture before the New Zealand Society of Civil Engineers, March* 11*th*, 1915.) PEE.3.c.1a.)

F.b.35.—Petrol and Petroleum Spirits. By W. E. Guttentag. Edward Arnold. 1918.

F.c.22.—Aero Engines and their Lubrication. Published by C. C. Wakefield and Co. 1938.

F.c.23.—*Lubrication. By W. R. Ormandy.* (*Lecture before Institution of Mechanical Engineers, March* 25*th*, 1927.) (PF.1.b.2.)

F.c.24-26.—*Survey of Expert Opinion on Lubricating Oils. New York University.* 1930. (Three pamphlets, entitled respectively):—

Members of the Society of Automotive Engineers.

Opinions of Automotive and Industrial Executives.

Qualities and Sources. (PF.1.b.3-5.)

*G.b.42.—Stobbs' Wire and Sheet Gauge Tables and Metal Calculator. By Thomas Stobbs. E. and F. Spon, Ltd. 1916.

*G.b.43.—Chart of the Elements. Metallic Compositions Co. (Undated.) (Contains data regarding Minerals and Metals.)

G.e.C.18.—*Nickel Alloys in Aeronautical Engineering. By H. G. Hall. Mond Nickel Co., Ltd.* 1937. (File Case.)

G.e.C.19.—*Some Applications of "Ni-Resist" in Automobile Construction. Mond Nickel Co., Ltd.* (*Undated.*) (File Case.)

G.e.J.22.—*The Mineral Deposits of Burma. By G. de P. Cotter. Govt. Printing Works, Rangoon.* (PG.7.b.3.)

H.a.11 and 12.—*Prevention of Corruption Act.* 1906 *and* 1916 *Eds. Eyre and Spottiswoode.* (PH.1.a.1 and 2.)

I.e.22.—Physical and Chemical Constants and some Mathematical Functions. G. W. C. Kaye and T. H. Laby. Longmans, Green and Co. 1911.

L.a.43.—R.A.F. Flying Training Part 2. (A.P. 928.) H.M.S.O. 1924.

M.c.60.—*Wireless Telephony on Aeroplanes. By Major C. E. Prince. Institution of Electrical Engineers.* 1920. (Y.18/20.b.)

P.b.11 and 12.—*Patent and Designs Act.* 1907 *and* 1919 *Eds. Eyre and Spottiswoode.* (PH.1.a.1 and 2.)

R.e.49.—The Mastery of Air. Sir Isaac Pitman and Co. 1922.

R.f.77 and 78.—*Aviation Atlas* (*2nd Edition*, 1931, *and* 4*th Edition*, 1935-1936). *Gulf Refining Co.* (Y.11.iv.a. 12a and 15a.)

R.f.79.—*Record du Monde d'Altitude avec un Bristol Pegasus Moteur en Étoile refroidi par Air.* (*Uwins*, 1931.) *Bristol Aeroplane Co.* (Y.11.a.12b.)

RR.113.—A Manual of Petrol Motors and Motor Cars. By F. Strickland. Chas. Griffin and Co. 1914.

RR.114.—Petrol Motors and Motor Cars. By T. Hyler White. Longmans Green and Co. 1909.

**S.a.123.—Ministry of Munitions of War: List of Staff and Distribution of Duties. 1918.

S.a.124.—War in the Air. Edited by Sir John Hammerton. Amalgamated Press. (Undated.)

T.b.78.—My Flight to the Cape and Back. By Sir Alan Cobham. A. and C. Black. 1926.

T.b.79.—Australia and Back. By Sir Alan Cobham. A. and C. Black. 1926.

X.c.B.23.—British Standard Glossary of Aeronautical Terms. British Standards Institution. 1933.

*X.e.48.—The British Aircraft Industry. Society of British Aircraft Constructors. 1939.

J. LAURENCE PRITCHARD, *Secretary and Editor*

THE STRESSES DEVELOPED IN SECTIONS SUBJECTED TO BENDING MOMENT.

An Extension of the Theory of Bending to include the Effect of Stress-Strain Characteristic.

By J. L. Beilschmidt, F.R.S.A., A.R.Ae.S.

It is a well known fact that the (simple) Engineers' Theory of Bending is based on the assumption of the Law of Proportionality, that is to say that the stress-strain characteristic of the material concerned is a straight line, and it accordingly follows that stress values determined by the formulæ: $M\bar{Y}/I$, for the stress at any station on the section, and M/Z for the maximum fibre stress can be correct only when the M/Z value lies within the limit of proportionality of the material.

There are a number of aircraft materials which have a very low limit of proportionality as compared with the ultimate strength of the material and in many cases the limit of proportionality as a definable point simply does not exist at all. It is therefore apparent that, since the above formulæ cease to apply as soon as any fibre in the section is working at a stress which is above the proportionality limit. it is quite useless, in the case of the majority of materials, to quote the M/Z stress as a criterion of rupture of the fibre of maximum stress against the ultimate strength of the material.

Nevertheless it is standard practice in the course of stress analysing aircraft structural components to quote reserve factors on the ultimate or yield stress of the material in respect of an M/Z value, which may well be above the limit of proportionality, even supposing that the latter does exist. It will be shown that although at first sight apparently inconsistent and incorrect, that given certain conditions, the practice so applied will not give rise to any appreciable inconsistancy or accuracy and that a definite purpose can be fulfilled.

It is necessary to emphasise, however, that this method of stress determination is useful only if these conditions, which are determined mainly by the stress strain characteristics of the material, can be maintained, and that a definite danger does exist in the indiscriminate application of the above formulæ, one common result of which is a serious discrepancy of the strength and deflection values determined by stress calculations against the results obtained by mechanical test on the component.

Serious divergences of this kind, which are particularly noticeable in the case of certain light alloy components, were in the first place responsible for indicating the need for an extension of the bending theory to include the full effect of the non-linear stress strain characteristic suitable for practical application to the design of cast and forged airscrew blades, flying control levers, and other aircraft components.

General Consideration of the Effect of the Material Stress Strain Characteristic on the Moment of Resistance of a Section.

It has already been indicated that by defining the stress at any given point on the section by $M\bar{Y}/I$, or the stress in the most highly stressed fibres by M/Z, it is implicit that the stress is proportional to the strain, from which it follows that, if in considering a rectangular element of width of the section, the stress

values so defined be erected normal to the section, all points will be on a plane surface between the maximum stress values, and passing through the neutral axis of the section. It is likewise apparent that a curved stress strain characteristic of the material will cause the points to be situated on a curved surface (Fig. 1).

The moment of resistance of the element of section is given by the product of the width of the element and the moment of the area of the stress distribution diagram about the neutral axis. In the case of the linear stress distribution, the area is represented by the triangles $a\ b\ o\ b'\ a'$, and in the case of non-linear distribution the area is defined as $a\ b_2\ c\ o\ c'\ b_2'\ a'$. Thus for any given maximum stress M/Z, the effect of convex curvature of the characteristic is to cause an increase in the moment of resistance, or conversely, if to represent a given moment of resistance the moment of the areas in the two cases be made equal, the maximum stress in the outermost fibres will be less than the apparent M/Z value by an amount which increases with increasing curvature of the stress strain curve.

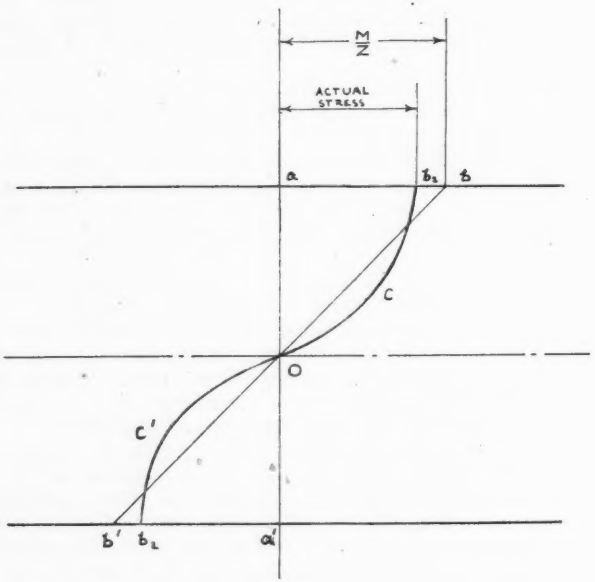

FIG. 1.

In the case of a large number of materials the yield point or the 1 per cent. proof stress is usually fairly well defined, and further, as for example in the case of steel, this stress corresponds to, or is slightly more than 75 per cent. of the ultimate tensile stress. The specified proof factor for aircraft components is 75 per cent. of the ultimate factor, consequently in the case of a metal having a proof stress of at least 75 per cent. of the ultimate tensile stress, it would be possible to design the beam with the calculated M/Z value corresponding to the specified ultimate factor made equal to the ultimate tensile strength of the material (Reserve Factor = 1.0), whereas the realised stress corresponding to the proof factor would nevertheless still be within, or for practical purposes near enough within, the proportionality limit. Nevertheless, this case, and even though the majority of the steels and many other materials come within its scope, must be

regarded as a special case in which it is permissible to work on ultimate M/Z and still have the working stress within the range of the linear law and to which therefore the M/Z is applicable. The ultimate M/Z stress so calculated has of course no relation to the fictitious, or apparant M/Z value at which absolute failure of the fibres in tension will occur and the act of comparing the ultimate M/Z stress with the ultimate strength of the material, and consequently, the ultimate reserve factor will, as far as the absolute failure is concerned, be quite meaningless.

For the purpose of convenience in routine stress work in order to obviate the necessity of calculating both ultimate and proof stresses and owing to the convention of quoting the reserve factor corresponding to the specified ultimate factor only, an attempt is often made to extend the practice as noted above to materials which have a proof stress less than 75 per cent. of the ultimate by the simple expedient of using in place of the ultimate stress of the material an " allowable stress " which is approximately four thirds of the proof stress, so that the M/Z calculated on the specified ultimate factored load, although again having no relation to the actual stress which would be realised under test conditions, will nevertheless, if within the limiting allowable stress mentioned, ensure compliance with the proof factor requirements.

Due consideration of these facts makes it evident that it is reasonable to expect satisfactory results from this method of designing bending sections only if the proof stress of the material is fairly high, which is to say that a considerable portion of the stress strain curve approximates to a straight line. Moreover, it is not possible to extend the practice of working on the basis of the linear law indefinitely since there are in existence certain alloys, notably the magnesium alloys which have a stress strain characteristic which is curved almost right from the origin, and any attempt to cater for the resulting relatively low proof stress (example, D.T.D. 289, proof 4.5-5 tons/in.^{2}, ultimate 13-16 tons/in.^{2}) by means of adjusting the allowable stress for the ultimate factored load on the basis of the linear law, a pessimistic and entirely erroneous assessment of the strength of components designed on this basis will be the result.

As an example of what can occur, a circular ring section member, $2\frac{1}{2}$ in. O.D.; $1\frac{1}{8}$ in. I.D. in magnesium alloy to specn. D.T.D. 289 on being subjected to a bending test developed an apparent M/Z of 14-17 tons/in.^{2} without causing any noticeable permanent set, this being increased to 28 tons/in.^{2} without causing absolute failure of the fibres, and this for a material having strength properties as stated in preceding paragraph.

Use of Allowable Ultimate Stress Adjusted to Cover Proof Stress Requirements on Basis of Non-Linear Stress-Strain Characteristic.

The convenience and expediency as far as practical purposes are concerned of the use of adjusted ultimate allowable stress to cover proof stress requirements has been noted. It is not the purpose of the present notes to argue the logic of the use of fictitious or apparent ultimate stress values which differ from the failing stress of the material, the fact remains that the practice is widely applied in the course of aircraft strength calculations and it therefore seems legitimate and sensible to adhere to the practice when extending the theory of bending to the non-linear law in view of the simplification and reduction of labour involved in making stress calculations which may thereby be achieved.

When an attempt is made to analyse the stress-strain curves of a number of different alloys it is apparent that the curves can, in the majority of cases up to at least 75 per cent. of the ultimate stress, be considered as conforming to an equation of the form $a x^n$. On the basis that the truth of this statement can be upheld under actual working conditions with at least a reasonable degree of accuracy, this being an aspect of the matter which is discussed further on, then

it follows that the Moment of Resistance of a rectangular strip element of section of width δB, and depth D is given by

$$\delta B \int_{-D/2}^{D/2} a x^{n+1} dx$$

the solution of this being

$$\delta B \times \frac{a\,(D/2)^{n+2}}{n+2}$$

Denoting this term by m, then the Moment of Resistance of a section consisting of such rectangular strip elements is given by Σm, the evaluation of which in the case of symmetrical rectangular channel, box and I-sections is capable of simple solution, thus for the solid rectangle,

$$\text{Moment of Resistance} = B \int_{-D/2}^{D/2} a x^{n+1} dx$$

which for the symmetrical box, channel or I-section is extended to

$$\text{M. of R.} = B \int_{-D/2}^{D/2} a x^{n+1} dx - b \int_{-d/2}^{d/2} a x^{n+1} dx$$

the solution of which is

$$\text{M. of R.} = \frac{a\,[B\,(D/2)^{n+2} - b\,(d/2)^{n+2}]}{(n+2)}$$

For the solid circle

$$\text{M. of R.} = 2 \int_{-\pi/2}^{\pi/2} R d\theta \left\{ \frac{(a R^{n+1} \cos^{n+1} \theta)}{(n+1)} \right\} (R \cos^2 \theta) \left\{ \frac{(n+1)}{(n+2)} \right\}$$

the solution of which, by introducing the gamma function is

$$\text{M. of R.} = \left\{ \frac{(4 a R^{n+3})}{(n+2)} \right\} \left\{ \frac{\Gamma\,(2+n/2)\,\Gamma\,(\tfrac{1}{2})}{\Gamma\,(2.5+n/2)} \right\}$$

which likewise can be extended to apply to the hollow tubular sections.

Simple analytical treatment as indicated above cannot of course always be applied in the cases of certain sections, which may tend to become awkward, in which case it might be more expedient to carry out the integration by other methods.

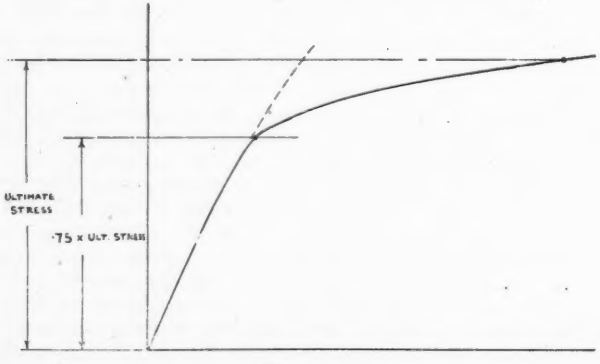

FIG. 2.

It will be noted that for all conditions of stress distribution in the case of metals, the power n will be either unity (corresponding to linear law) or less than unity. Putting $N=1$ in any of the above equations will give the Moment of Resistance of the section for the condition of the linear stress distribution, which is equivalent to $Z \cdot p$; where Z is the section modulus and p is the max. fibre stress.

If we divide the M. of R. which would occur in the case of linear stress distribution into the actual M. of R., the resulting ratio is also the ratio by which the apparent or M/Z stress exceeds the actual stress.

From the foregoing, it follows that this ratio is dependent entirely on, (*a*) the stress-strain characteristic of the metal as defined by the values a and n, and (*b*) by the geometrical characteristics of the section. In the following text, this ratio will be referred to as the Form Factor and will be something greater than unity, approaching unity at the linear condition.

Since the form factor is represented by the fractions, M. of R. actual/M. of R. linear distribution, it also follows that if the ultimate stress be arbitrarily considered as being a point on a fictitious extension of the stress strain curve (Fig. 2), which is made to conform to the same ax^n as the lower part of the curve it is merely equivalent to multiplying both numerator and denominator by a constant, 1/0.75 which cancels out before the final term is reached and the form factors calculated in the manner described are therefore unaltered.

Thus assuming that the proof load is 0.75 of the specified ultimate factored load and M is a bending moment corresponding to the specified ultimate factor, then

$$\text{Actual max. fibre stress under proof load} = \frac{0.75\ M}{(Z \times \text{form factor})}$$

and if

$$\frac{M}{(Z \times \text{for factor})} \gtrless \text{ultimate stress of the metal,}$$

then automatic compliance with the proof factor requirement is ensured.

It is essential to note, however, that the stress as calculated from the latter formula is a fictitious value and does not represent the modulus of rupture of that particular section and material.

Estimating Modulus of Rupture.

It will be shown that although the stress-strain curves of most metals can be closely approximated by a curve of the type ax^n, it is not possible to line up all the ordinates in this manner absolutely, thus, for example, the characteristic of A.Z. 31 (D.T.D. 289), as indicated in Fig. 3, which if taken as a whole right up to the ultimate stress point can be fitted fairly well with such a curve if the index n be taken as 0.215 throughout. That portion of the curve up to the proof stress point is far more easily fitted by such a curve when n is taken as 0.875. This is clearly shown by the left hand illustrations in Fig. 3.

The suggestion is that, where necessary, a different value of n should be used when estimating stresses which lie on that portion of the curve between the proof and the ultimate rupture stress, to the value of n used for covering the proof stress condition as outlined above.

Consideration of the Degree of Accuracy Attainable.

It is necessary to consider the extent to which ordinates of stress strain curves as obtained from tensile test plottings can be aligned with curves of the type specified with the attainment of an accuracy sufficient to come within the limit of experimental error.

The stress strain characteristic of some casting and forging alloys have been analysed with a view to evaluating the index at various points on the curves.

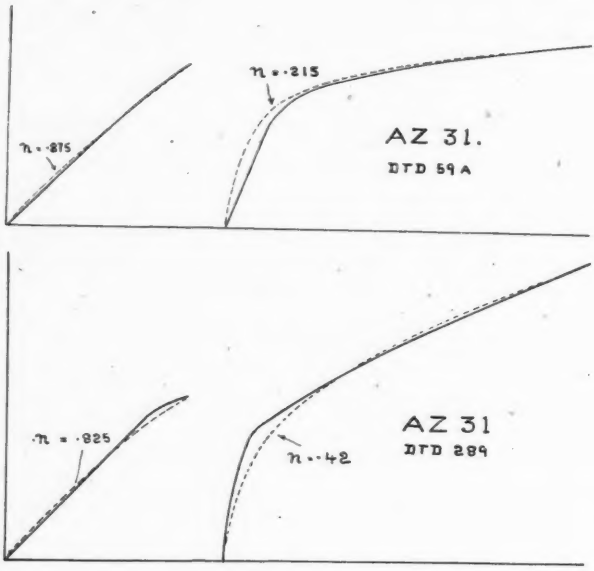
n = ·875
n = ·215
AZ 31.
DTD 59A
n = ·825
n = ·42
AZ 31
DTD 289

FIG. 3.

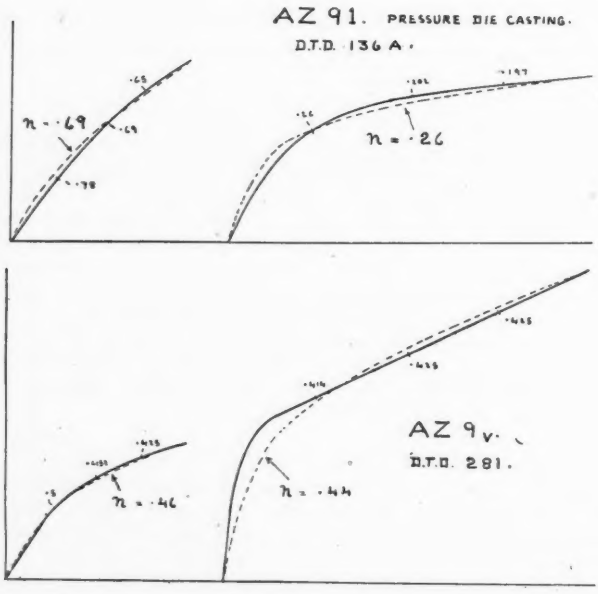
AZ 91. PRESSURE DIE CASTING.
D.T.D. 136A.
n = ·69
n = ·26
AZ 9 V.
D.T.D. 281.
n = ·46
n = ·44

FIG. 4.

It is at once apparent that the index n is seldom constant for any considerable portion of the curve, the tendency being for it to commence decreasing gradually some way before the proof stress is reached up to a region where the metal commences to flow and so cause a rapid drop in n. From then onwards n decreases at a more or less gradual rate until failure of the material occurs. Indeed, in the case of some of the alloys this effect is so very pronounced as to give the effect of two separate curves, each of which has a fairly constant value of n in itself, but differing considerably one from the other. This leads to the suggestion as to whether two separate cases, and attaching a constant value of n to each, it would be possible to achieve results sufficiently accurate to be of practical value. That is to say, it is necessary to ascertain whether the curve taken as a whole can be reasonably fitted with a curve for which n is constant.

This has been investigated and the conclusion to be drawn is that this procedure can be applied with a fair measure of success. The results of the investigation on some typical alloys is given in Table I, which are further amplified by the stress strain curves as shown in Figs. 3, 4 and 5.

TABLE I.

ANALYSIS OF THE STRESS-STRAIN CHARACTERISTICS OF SOME LIGHT ALLOYS.

		Characteristic for Proof Stress.	Characteristic for Ultimate Stress.
A.Z. 31 D.T.D. 59A	Station .25 X, $n=$	.95	.221
	,, .5 X, $n=$	.875	.211
	,, .75 X, $n=$	.87	.271
	Mean curve, $n=$	.875	.215
A.Z. 31 D.T.D. 289	Station .25 X, $n=$	.92	.4
	,, .5 X, $n=$	.846	.45
	,, .75 X, $n=$	.683	.485
	Mean curve, $n=$	.825	.42
A.Z. 91 D.T.D. 136A	Station .25 X, $n=$	.78	.262
	,, .5 X, $n=$	.69	.202
	,, .75 X, $n=$	.65	.197
	Mean curve, $n=$	.69	.26
A.Z. 9V D.T.D. 281	Station .25 X, $n=$	.5	.414
	,, .5 X, $n=$	.452	.425
	,, .75 X, $n=$	.425	.425
	Mean curve, $n=$	.46	.44
A.M. 503 Wrought D.T.D. 142	Station .25 X, $n=$	.724	.209
	,, .5 X, $n=$	.446	.248
	,, .75 X, $n=$	.367	.305
	Mean curve, $n=$	.51	.245
A.Z.M. Extruded D.T.D. 886	Station .25 X, $n=$	.935	.143
	,, .5 X, $n=$	.905	.177
	,, .75 X, $n=$	.775	.2155
	Mean curve, $n=$	.85	.18

On the right hand side of the figures are shown stress strain curves taken up to the failure, whereas the left hand view shows in each case the portion of the curve below the proof stress drawn out to a larger scale. Values of n along the curve are noted, and in order to show a visual comparison, the suggested constant n curve is shown superimposed upon the actual curve.

It will be apparent from an inspection of these curves that by proceeding in the manner suggested and treating the proof stress and the ultimate condition separately, it is possible to obtain a very fair degree of alignment of the mathematical with the experimental curves.

Even in the case of the D.T.D. 289 alloy, in which the alignment with the test result curve is among the least satisfactory, it can be shown (by taking moments of the area about the Y axis of the curve) that the error is nil for the rectangular strip element and 2 per cent. for the solid circle, which may, for practical purposes, be considered as representing the condition of least accuracy.

EFFECT OF THE MATERIAL STRESS-STRAIN CHARACTERISTIC ON THE POSITION OF NEUTRAL AXIS.

For sections which are symmetrical about the axis which is at right angles to the plane of bending, and providing that the shape of the stress distribution curve is the same on the compression side as it is for the tension side, then the actual

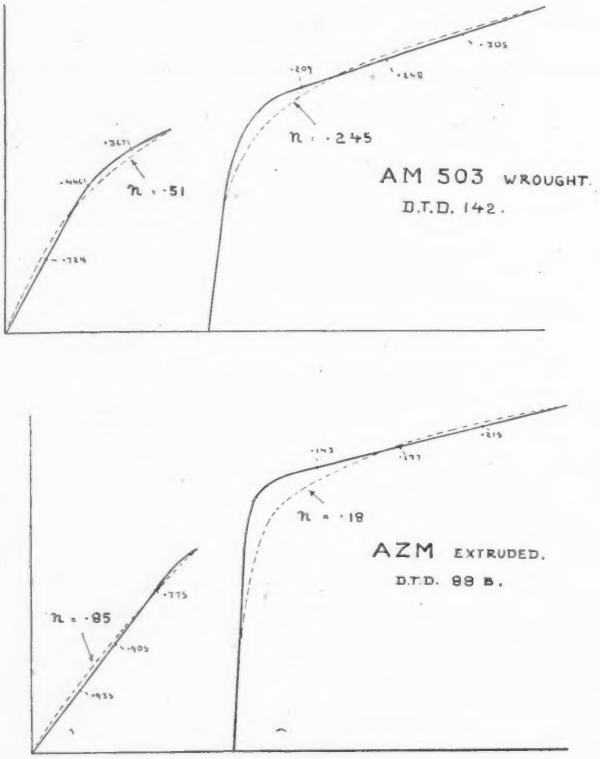

FIG. 5.

stress will be of the same magnitude both in the outermost tension and compression fibres, and will be simply equivalent to M/ZF. But in the case of materials having differing stress-strain characteristics for tension and compression, and also in the case of non-symmetrical sections, the simple formula is no longer applicable, although the form factors as such do still represent the conditions both on the tension and compression side, taking each separately.

The axial load in the beam for each rectangular strip element of section can be represented as the values produced by the product of the width of the strip and the area enclosed by the curve formed by the points of the stress ordinates (the area a, b, c, o in Fig. 7, for example). Likewise, and for any section this load corresponds to the volume represented by the summation of the products of the stress ordinates and the area of the particles of which the section can be considered as being composed.

In order to satisfy the fundamental law of the equilibrium of forces, it is necessary for the axial loads above and below the neutral axis to be equal in magnitude, and it is this condition which actually determines the position of the neutral axis. If the value of a is the same for tension as for compression, and also at the same time that n is unity (identical linear stress-strain characteristic for tension and compression), then the neutral axis will coincide with the geometrical centroid of the section. It can, however, be said that only when the material characteristics are thus that the neutral axis and the centroid must of necessity be coincident in the case of a non-symmetrical section. If a, or n, are different on one side of the section only, or if in the case of a non-symmetrical section, n is other than unity, then the neutral axis will no longer be on the geometrical centroid of the section.

Defining the Neutral Axis Position for Non-Symmetrical Sections and Dissimilar Tension and Compression Material Characteristics.

The axial load on the one side of the neutral plane of the rectangular strip can be represented as

$$P=\delta b\int_0^{D/2} ax^n \,.\, dx=\delta\frac{(ax^{n+1})}{(n+1)}$$

which is simply the value obtained by multiplying the width of the strip by the area of the stress distribution curve.

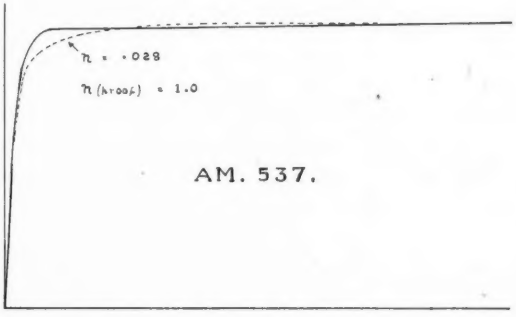

Fig. 6.

In compliance with the rule of the equilibrium of forces which, as already stated, requires that the force above and below the neutral axis be equal, it must be,

$$P=\bar{P}$$

or

$$\frac{(ax^{n+1})}{(n+1)}=\frac{(\overline{ax}^{n+1})}{(\bar{n}+1)}$$

where the " bar " quantities are intended to refer to the values on the other side of the neutral plane, *e.g.*, the compression side for example.

Solving the above

$$\frac{(ax^{n+1})}{(n+1)} = \frac{\bar{a}}{(\bar{n}+1)}$$

$$\therefore x^{n+1} = \left\{\frac{(n+1)}{(\bar{n}+1)}\right\} \times \left(\frac{\bar{a}}{a}\right)$$

$$\frac{X}{\bar{X}} = [\ \{\ (n+1)/(\bar{n}+1)\ \}\ (\bar{a}/a)]^{1/(n+1)}$$

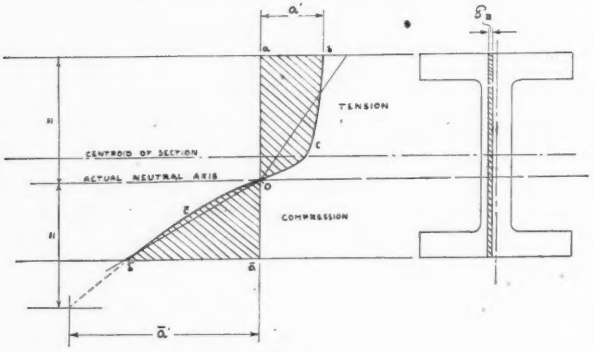

FIG. 7.

Showing type of stress distribution due to dissimilar tension and compression characteristic.

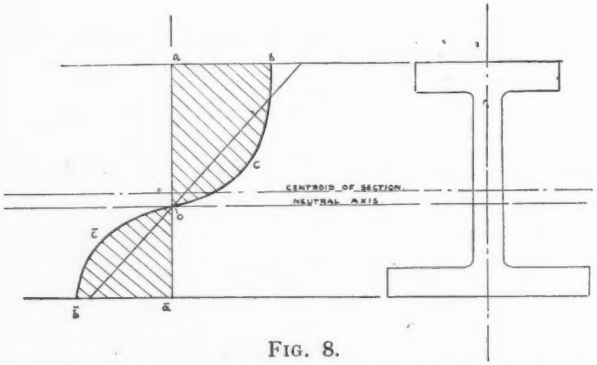

FIG. 8.

Non-linear stress distribution in asymmetrical section.

Thus given the values a and $\bar{a}$, n and n, the location of the neutral axis in the case of the rectangular strip, Fig. 7, on a plain rectangular section is simply defined, *e.g.*, taking the full depth of the section as $X+\bar{X}=D$, then

$$X = \left(D\ \frac{X}{\bar{X}}\right) \Big/ \left(\frac{X}{\bar{X}}+1\right)$$

For the I, channel, box, L and T sections, the expression for P is of a similar form to that given for the M. of R., thus :—

$$P = B\int_0^X ax^n dx - b\int_0^{X^1} ax^n dx$$

where X' corresponds to the distance from the neutral axis of the inner edge of the flange, *e.g.*, in the case of a section symmetrical about the neutral axis, it would be, $X'=d/2$.

The above expression is likewise capable of the solution

$$P=\frac{a\,(BX^{n+1}-bX'^{n+1})}{(n+1)}$$

For the case of the tubular sections and the solid circle, the solution can be

$$P=(R^{n+2}-r^{n+2})\,.\,\frac{\{\,\Gamma\,(1.5+n/2)\,\Gamma\,(\tfrac{1}{2})\,\}}{\{\,\Gamma\,2+n/2)\,n+1\,\}}$$

again introducing the " gamma " functions.

TABLE II.

FORM FACTORS FOR MOMENT OF RESISTANCE.

RECTANGULAR, I, [AND BOX SECTIONS.

$\frac{b}{B}$	$\frac{d}{D}$	n								
		1.0	.9	.8	.7	.6	.5	.4	.3	.2
.0	0.25 0.5 0.75	1.0	1.035	1.072	1.11	1.15	1.2	1.25	1.31	1.36
0.25	0.25	1.0	1.035	1.07	1.10	1.15	1.2	1.245	1.3	1.355
	0.5	1.0	1.034	1.068	1.10	1.142	1.185	1.23	1.28	1.33
	0.75	1.0	1.05	1.067	1.10	1.14	1.18	1.226	1.272	1.325
0.5	0.25	1.0	1.034	1.068	1.11	1.152	1.19	1.238	1.289	1.342
	0.5	1.0	1.03	1.06	1.095	1.13	1.168	1.22	1.25	1.3
	0.75	1.0	1.03	1.055	1.086	1.115	1.151	1.19	1.255	1.272
0.75	0.25	1.0	1.033	1.067	1.10	1.143	1.187	1.23	1.28	1.33
	0.5	1.0	1.03	1.057	1.085	1.118	1.15	1.186	1.245	1.265
	0.75	1.0	1.022	1.042	1.064	1.086	1.115	1.145	1.172	1.205
1.0	0.25	1.0	1.035	1.065	1.10	1.14	1.18	1.23	1.27	1.32
	0.5	1.0	1.025	1.048	1.073	1.10	1.13	1.16	1.19	1.225
	0.75	1.0	1.016	1.025	1.036	1.047	1.068	1.082	1.093	1.11

TABLE III.

FORM FACTORS FOR MOMENT OF RESISTANCE. TUBULAR SECTIONS.

$\frac{d}{D}$	n								
	1.0	.9	.8	.7	.6	.5	.4	.3	.2
Solid circle	1.0	1.046	1.098	1.15	1.21	1.268	1.35	1.415	1.51
.25	1.0	1.046	1.095	1.145	1.205	1.266	1.345	1.41	1.5
.5	1.0	1.04	1.085	1.13	1.178	1.236	1.3	1.357	1.435
.75	1.0	1.032	1.062	1.1	1.14	1.174	1.225	1.265	1.32
Thin walled tube	1.0	1.022	1.048	1.073	1.1	1.13	1.162	1.196	1.235

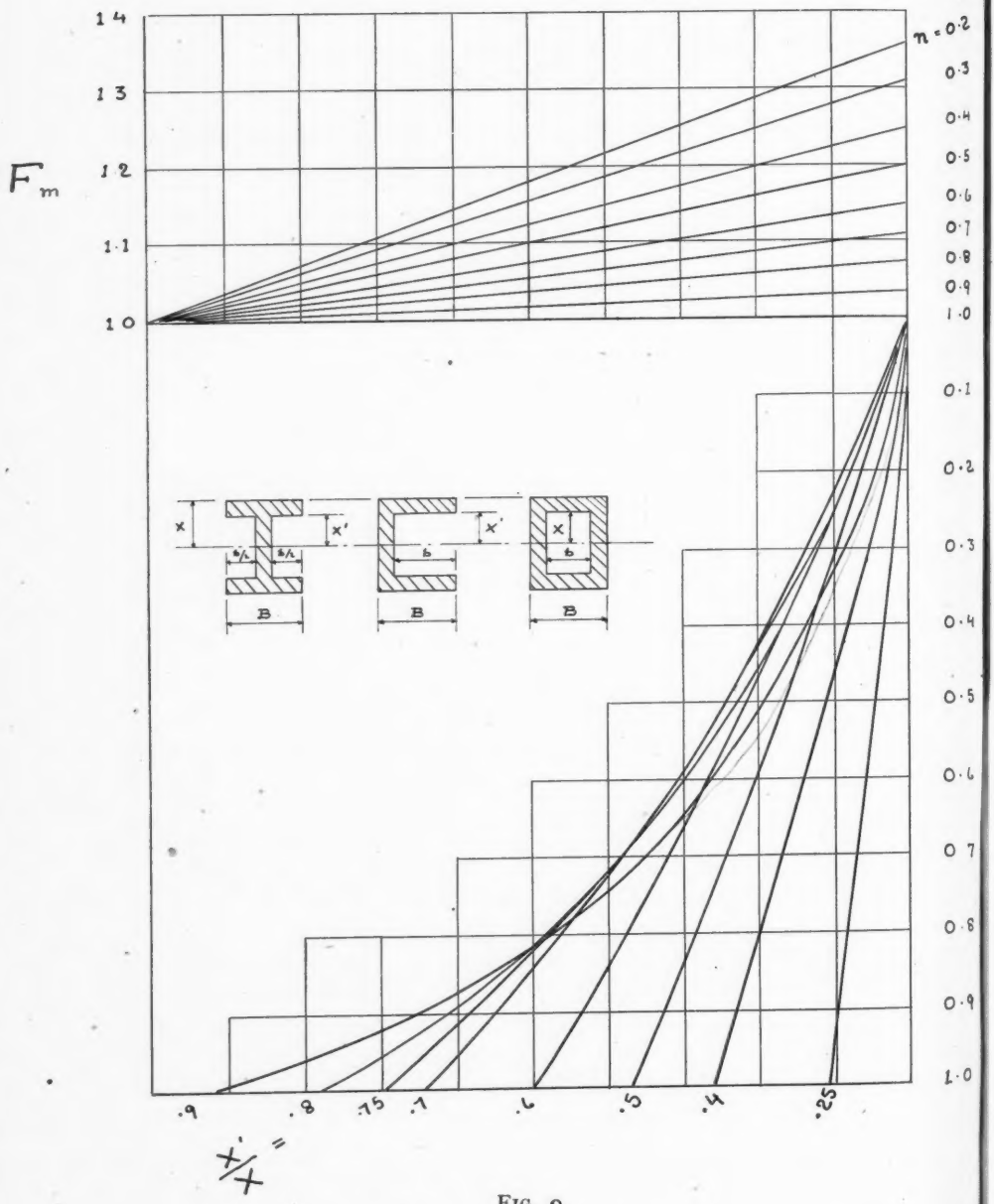

FIG. 9.

Form factors for moment of resistance.

Now since in the case of the rectangular strip element when $n=1$, the force P is also equal to $\delta b \times aX^2/2$, the term also representing the first moment of area of the section on one side of the neutral axis, it can also be stated for any form of section, and any value of n,

$$P=\Sigma\delta b\int ax^n dx=F_v \times AY$$

where F_v is a form factor for axial force, or stress-strain volume and is definable as being the ratio of the axial force stress-strain volume against the stress-strain volume when n is unity, and exactly as in the case of the form factors for moment will be entirely dependent on the value of n, and on the geometrical properties of the section concerned.

The equation for the equilibrium of forces can now also be written :—

$$F_v aAY=\overline{F}_v\bar{a}\overline{AY}$$

where AY and $\overline{AY}$ denote the first moment of the area, respectively, of the section above and below the neutral axis.

The first moment of area can also be written in the form

$$AY=CX^2$$

The equation then becomes

$$F_v aCX^2=\overline{F}_v\bar{a}\overline{C}\overline{X}^2$$

so that

$$\frac{X}{\overline{X}}=\{(F_v aC)/(\overline{F}_v\bar{a}\overline{C})\}^{\frac{1}{2}}$$

TABLE IV.

FORM FACTORS FOR AXIAL LOAD (STRESS STRAIN VOLUME).
RECTANGULAR, I, [AND BOX SECTIONS.

$\frac{b}{B}$	$\frac{d}{D}$	n								
		1.0	.9	.8	.7	.6	.5	.4	.3	.2
0	0.25 0.5 0.75	1.0	1.051	1.112	1.178	1.25	1.335	1.428	1.54	1.668
0.25	0.25	1.0	1.05	1.105	1.169	1.235	1.131	1.398	1.5	1.615
	0.5	1.0	1.045	1.1	1.158	1.224	1.3	1.38	1.475	1.585
	0.75	1.0	1.045	1.1	1.16	1.23	1.297	1.386	1.482	1.6
0.5	0.25	1.0	1.05	1.1	1.16	1.22	1.29	1.365	1.458	1.56
	0.5	1.0	1.081	1.088	1.14	1.192	1.26	1.321	1.4	1.491
	0.75	1.0	1.04	1.085	1.138	1.19	1.25	1.32	1.402	1.498
0.75	0.25	1.0	1.051	1.052	1.153	1.208	1.272	1.342	1.421	1.51
	0.5	1.0	1.035	1.075	1.118	1.158	1.215	1.248	1.321	1.385
	0.75	1.0	1.028	1.061	1.11	1.135	1.18	1.225	1.285	1.35
1.0	0.25	1.0	1.045	1.088	1.135	1.188	1.242	1.305	1.37	1.441
	0.5	1.0	1.027	1.055	1.087	1.115	1.147	1.18	1.22	1.256
	0.75	1.0	1.005	1.02	1.037	1.046	1.06	1.072	1.09	1.11

Since the value of C and $\overline{C}$, and F_v and $\overline{F}_v$ are in themselves dependent on X and $\overline{X}$, which are the unknown quantities, it is unfortunately not possible to solve the expression other than successive approximation, or trial and error methods. In order to facilitate this, the values for C against D/d and B/b for

the rectangular type of sections, are given in Table VI, which are supplemented by the curves, Fig. 13.

Indeed, for routine stressing purposes it would be advantageous and expedient to combine the F_v and C values in a single set of curves and it is intended to produce these curves in due course. It will then be necessary to refer to a single form factor applicable to the value of n, and the D/d (or $X/\overline{X}$) and B/b ratios in when at present reference must be made to two values, and the true form factor has been kept distinct.

Values for the plain form factors for stress-strain volume, F_v, are given in the appended tables, IV and V, supplemented by the curves in Figs. 11 and 12.

TABLE V.

$\frac{d}{D}$	n								
	1.0	.9	.8	.7	.6	.5	.4	.3	.2
Solid circle	1.0	1.07	1.148	1.232	1.331	1.44	1.566	1.72	1.894
.25	1.0	1.065	1.138	1.222	1.315	1.418	1.534	1.675	1.83
.5	1.0	1.058	1.121	1.165	1.266	1.355	1.448	1.558	1.662
.75	1.0	1.046	1.105	1.146	1.21	1.275	1.35	1.438	1.57
Thin walled tube	1.0	1.03	1.078	1.12	1.181	1.228	1.289	1.357	1.427

F_v, Form Factors for Axial Load (Stress-Strain Volume) for Tubular Sections.

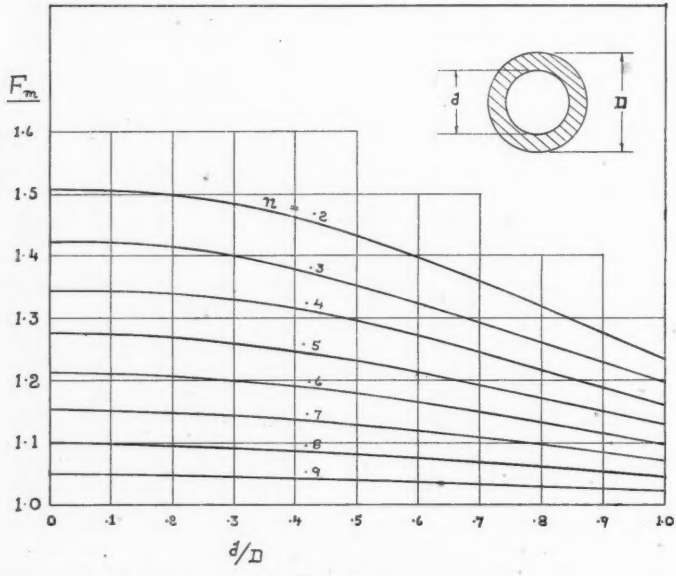

FIG. 10.

Form factors for moment of resistance.
Tubular sections.

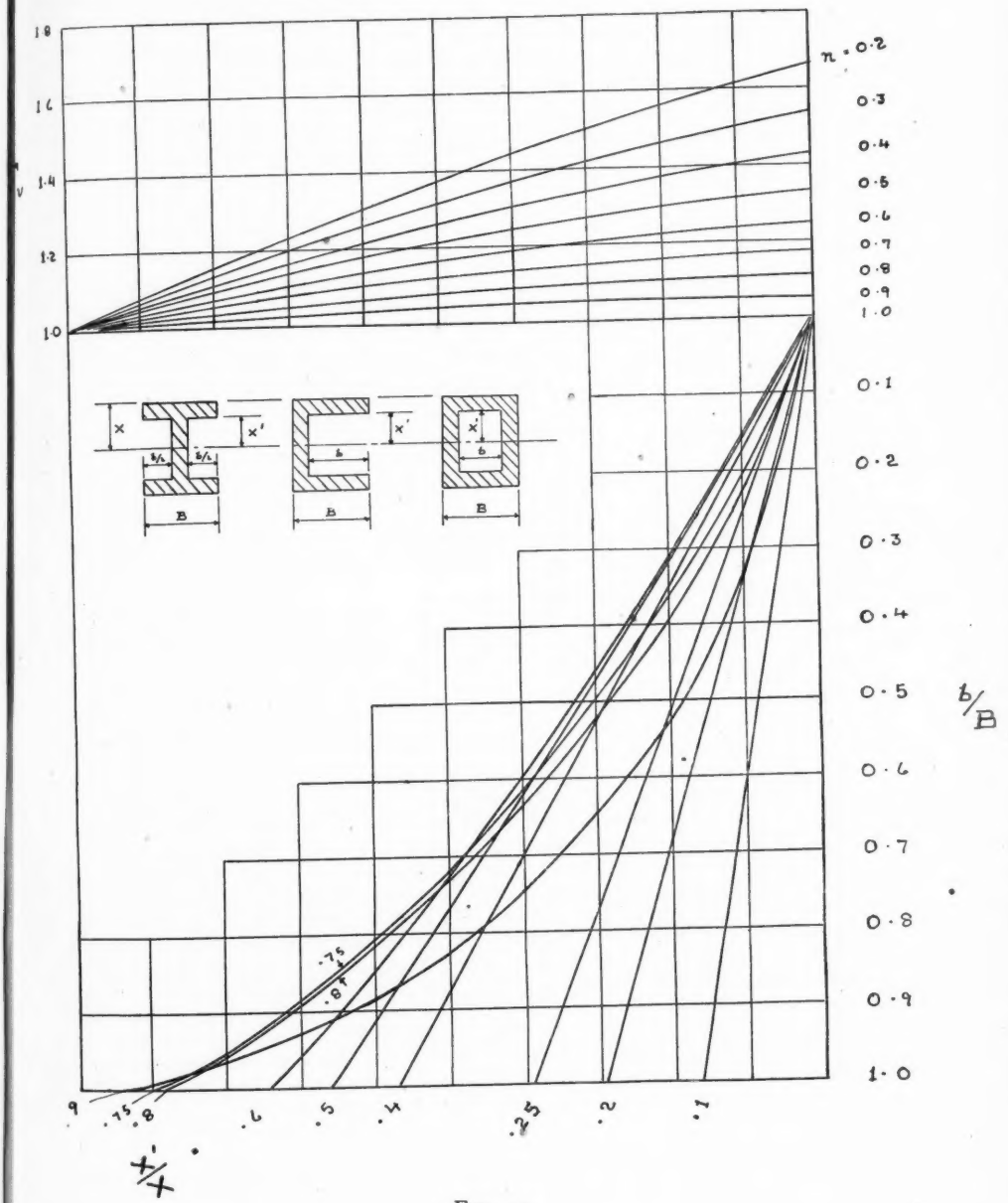

FIG. 11.

Form factors for axial load.

Once having found the position of the true neutral axis by determining the distance to the outermost tension and compression fibres, X and $\bar{X}$, the maximum tensile and compression stresses can then be calculated by using the Form Factors for Moment in the manner of the process already dealt with.

Thus taking X, n, a, I and F as referring to the tension side, and $\bar{X}$, $\bar{n}$, $\bar{a}$, $\bar{I}$ and $\bar{F}$ to the compression side, then

Maximum Tensile Stress in Fibre at distance X from N.A.

$$= \frac{M \cdot X}{(FI + \bar{F}\bar{I} \times \bar{a}/a \times X/\bar{X}^n)}$$

Maximum Compressive Stress in Fibre at distance $\bar{X}$ from N.A.

$$= \frac{M \cdot \bar{X}}{(\bar{F}\bar{I} + FI \times a/\bar{a} \times X/X^n)}$$

These two latter formulæ are in reality nothing more than a rearrangement of the formula for moment of resistance, which in the present case can be arranged and stated as,

$$\text{M. of R.} = \left(\frac{FI}{X} + \frac{\bar{F}\bar{I}}{\bar{X}} \times \frac{\bar{X}}{X} \times \frac{\bar{a}}{a} \times \frac{\bar{X}^n}{\bar{X}}\right) \cdot f_{\text{tension}}$$

or,

$$\text{M. of R.} = \left(\frac{\bar{F}\bar{I}}{\bar{X}} + \frac{FI}{X} \times \frac{X}{\bar{X}} \times \frac{a}{\bar{a}} \times \frac{X^n}{X}\right) \cdot f_{\text{compression}}$$

The presentation of all the preceding formulæ presupposes for their practical utilisation the possession of complete stress-strain characteristic data both for tension and compression.

The compressive stress-strain characteristic for most materials do usually differ, sometimes very considerably, from the tension characteristic. Materials tending toward the brittle have a compression characteristic which may be linear right up to the ultimate stress at which crushing or slipping of the planes occur, although the stress-strain rates, a and $\bar{a}$ for tension and compression can differ as, to name one example, occurs in the case of cast iron. Plastic materials which start to flow before the ultimate stress is reached do very often exhibit tension and compression characteristics which may be almost identical up to the yield stress zone in which the flow condition is established. As the plastic flow proceeds the basic tendency may again be identical or similar for both conditions. This is to say that the stress-strain characteristic based on unit cross sectional area may be the same but is modified by the change of area which is a simultaneous accompaniment of the extension or contraction.

Against this basic nominal characteristic, the reduction in area accompanying extension in the plastic region causes a falling off of the stress which is, of course, calculated on the basis of the original area of the specimen. During compression, a corresponding increase in cross sectional area takes place which, after the plastic flow has started, causes the stress to rise above the nominal basic line, the resulting curve developing approximately in accordance with a hyperbolic law.*

Materials possessing a high degree of ductility often have a characteristic which is linear over the entire plastic range as is evidenced by the characteristic stress-strain plots of certain light alloys, this being a feature particularly noticeable in the case of the A.Z. 91, A.Z. 31 and A.M. 537 alloys, the tensile characteristics of which appear in Figs. 3, 4 and 6.

There are also a number of materials which do not exhibit precisely the same tendency to behave in this manner.

Although the complete tensile characteristics for most commercial materials are readily available, this is not necessarily also the case for compression, and material compressive tests are not nearly so frequently carried out as are the tensile tests.

* Morley, Theory of Structures.

TABLE VI.

$b/B=$ \ $X'/X=$	.1	.2	.25	.3	.4	.5	.6	.7	.75	.8	.9	.95	1.0
.1	.999	.996	.9937	.991	.984	.975	.964	.951	.9439	.936	.919	.91	.9
.2	.998	.992	.9875	.982	.968	.95	.928	.902	.8938	.872	.838	.82	.8
.25	.9975	.99	.9844	.9775	.96	.9375	.91	.8775	.8598	.84	.7975	.775	.75
.3	.997	.988	.9812	.973	.952	.925	.892	.853	.8317	.808	.757	.73	.7
.4	.996	.984	.975	.964	.936	.9	.856	.804	.7876	.744	.676	.64	.6
.5	.995	.98	.9687	.955	.92	.875	.82	.755	.7195	.68	.595	.55	.5
.6	.994	.976	.9625	.946	.904	.85	.784	.706	.6634	.616	.514	.46	.4
.7	.993	.972	.9562	.937	.888	.825	.748	.657	.6073	.552	.433	.37	.3
.75	.9925	.97	.9532	.9325	.88	.8125	.73	.6325	.579	.52	.3925	.325	.25
.8	.992	.968	.95	.928	.872	.8	.712	.608	.5752	.488	.352	.28	.2
.9	.991	.964	.9437	.919	.856	.775	.675	.559	.4941	.424	.271	.19	.1
.95	.9905	.962	.9406	.9145	.848	.7625	.658	.535	.468	.3925	.23	.145	.05
1.0	.99	.96	.9375	.91	.84	.75	.64	.51	.439	.36	.19	.1	0

If the complete characteristic of a material is not available, but that nevertheless the yield stress and ultimate stress are known, and also if possible some data on the tendency and general shape of the characteristic, the curve can be approximated by the formula

$$n \text{ (approx.)} = \frac{(2\, fy^2/fu) + fy}{E\xi^2}$$

where fy is the yield stress taken fairly high, *e.g.*, the stress at which plastic flow is established rather than the stress at which the first deviation from the elastic portion of the characteristic becomes noticeable, commonly known as the proof stress, fu is the maximum fibre stress and can be the ultimate stress of the material. E = Young's Modulus, and ξ = elongation.

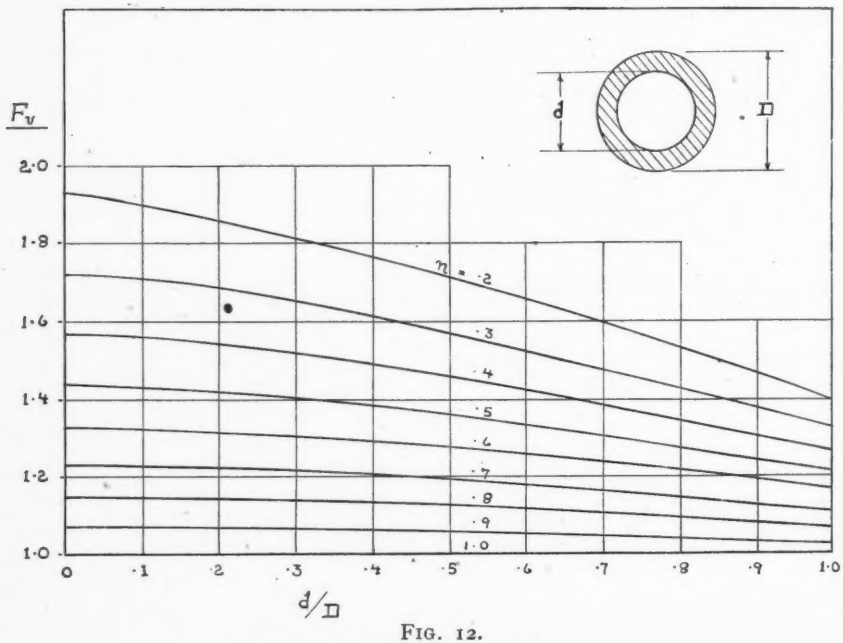

FIG. 12.

Form factors for axial load.
Tubular sections.

The formula is applicable to either the tension or the compression characteristic. In any case, however, it is to be clearly understood that this approximation formula is offered as a means of forming an estimate of the characteristic in the absence of complete data and is not intended as a substitute for such.

In order to facilitate the utilisation for practical purposes of the present extended theory of bending, it was decided to make some results available in a form convenient to use for stress checking sections having various geometrical proportions and for a variety of material characteristics. Accordingly the form factors have been calculated, using the various formulæ described in the foregoing text, for some of the commoner structural sections, *e.g.*, solid circle and solid rectangle, tubular, I, channel and box sections. The results have been tabulated and plotted and form an accompaniment to the present notes.

The Form Factors for Moment of Resistance for the solid rectangle, I, channel and box sections for various internal depth/external depth, and internal breadth/

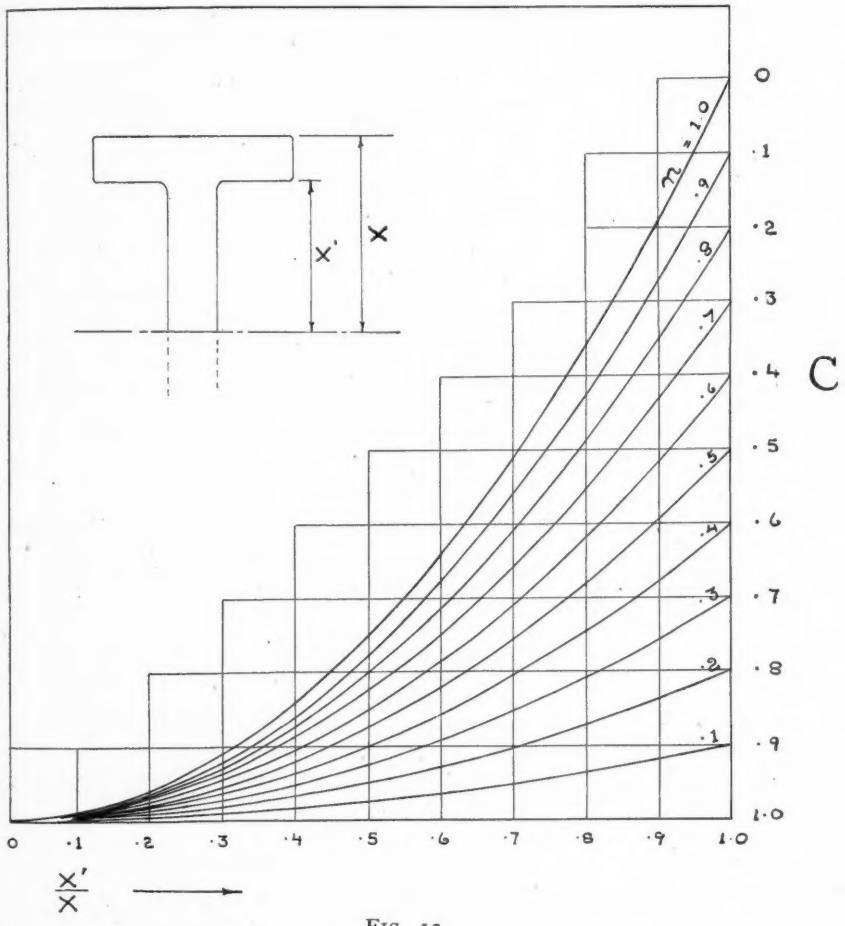

FIG. 13.

external breadth ratio are given in Table II, supplemented by the curves, Fig. 9. The corresponding Form Factors for Axial Load used for the neutral axis computation are given in Table IV, and curves, Fig. 11.

The Form Factors applicable to the tubular and solid circle are contained in Tables III and V, together with the corresponding curves, Figs. 10 and 12.

CONCLUSIONS.

The resistance offered to bending for given maximum tensile and compressive stress in the section, or conversely, the stresses developed for a given applied bending moment is entirely dependent on two factors alone, namely, the geometrical properties of the section concerned, and also the stress-strain characteristics of the material.

If the stress-strain characteristic is a curve which can be represented as being of some definite mathematical form, it is possible to base the calculation of stresses upon precise mathematical methods.

It can be shown that the characteristics of most metals conform to a curve of the form ax^n, in which case given values for a and n, and knowing the I and Z constants of the section, the matter is capable of simple solution.

It is noted that whereas the stress-strain characteristics, taken as a whole, can in most cases be satisfactorily fitted to a mathematical curve even though some local deviation occurs and it therefore becomes necessary, when analysing the characteristic, to deal with only that portion of the characteristic which is being used. Hence the need, for example, of treating the proof and the ultimate stress condition as separate cases to which different characteristics apply.

A method of correlating the section geometrical constants and the material stress-strain characteristics has accordingly been devised and takes the form of an extension of the engineers (simple) theory of bending.

An object throughout was to render the process and the results capable of straightforward application to routine stress calculation procedure. With this object in mind it was accordingly arranged to retain the familiar M/Z formula which gives the maximum fibre stress for the linear stress-strain law (Hook's law) adjustment of the stress for non-linear characteristics being obtained by means of correction factors, so-called " form factors."

One effect of a non-linear characteristic is to cause a displacement of the neutral axis with respect to the geometrical centroid of the section. A special type of correction factor, which can be described as a form factor for axial load, on stress volume, has been devised for the purpose of handling this effect.

The manner in which these form factors can be calculated is described and, furthermore, form factors have been calculated for a number of sections of various geometrical ratios which are commonly used for aircraft structural components and the results have been tabulated and plotted.

REVIEWS.

How to do Aircraft Sheet Metal Work.

C. Norcross and J. D. Quinn. McGraw Hill Book Company, Inc. New York and London. 1942. 15/-.

The authors of this book are Mr. Norcross, the Managing Editor of *Aviation*, and Lieutenant J. D. Quinn, the Officer in Charge of the Ground School, U.S. Naval Air Station, and who was formerly in charge of the Aviation Trades Centre, National Defence Training Programme, New York City. Both have made a close study of their subject matter and the result is a book that can be thoroughly recommended.

Although it describes American practice and uses American terms and methods the fundamental principles remain the same, and anyone who reads and understands what he reads will find in this book a mine of information which will bring him handsome royalties.

Chapter I begins with the statement " Blueprints are the language of the engineer," and is concerned from that moment with explaining the reading of simple blueprints. This chapter is extremely well illustrated with a number of blueprints. It is interesting to note the use made by the Douglas Aircraft Company of the " cut away " perspective sketch for helping the mechanic to visualise the detail parts involved as well as the entire assembly. The authors, however, are careful to point out that such drawings, useful as they are, do not in any way take the place of the blueprint.

Chapter II is a brief and useful survey of shop mathematics and Chapter III deals with working properties and standards of aircraft materials. The definitions in this chapter are more practical than mathematical as " Elasticity means stretch," " Tension is a force that pulls materials apart." In a future edition the authors would do well to consider re-writing the first three pages of this chapter, as looseness of definition can only lead to vagueness of ideas. This comment applies to definitions in other chapters and on a first reading may lead to an erroneous idea of the value of the book as a whole. For example, in Chapter IV it is stated " The entire circumference of a circle is divided into 360 units called degrees." The chapter headings which follow give an idea of the scope of the book, " How to measure," " Templates," " Aircraft sheet metal lay out," " How to cut sheet," " Files and how to use them," " Forming, stamping and hydraulic presses," " Drilling and how to do it," " How to rivet," " Jig assembly," " Skin fitting," " Spot welding," and " Shop projects."

Most of the chapters are illustrated by photographs of actual work on the jobs and all chapters contain a series of questions to test the student's understanding of the chapter he has just read.

Aircraft Instruments: Their Construction and Maintenance.

By J. Riley. N.A.G. Press, Ltd. 1942. 1/6.

Books on aircraft instruments are few and far between; those which do exist are mostly expensive and highly technical. The price, size and simplicity of this little volume will make it most acceptable to a young aircraft apprentice or A.T.C. Cadet. It bears all the signs, both good and bad, of having been compiled from

firms' catalogues: the good being represented by a profusion of admirable photographs and diagrams; the bad, by a tendency to skim lightly over points of theory in a way more calculated to baffle than to enlighten the student. A wide range of instruments is encompassed by this small book, which includes, in addition to the usual navigational and flying instruments, various types of gauges and thermometers, undercarriage position indicator, bubble sextant and aircraft watch.